Resonance Effect

of Collisional Drift Waves

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Abstract

In the inhomogeneous plasma there exists an effect which leads to a substantial increase of the amplification rate of the current driven instabilities: A resonant interaction between the drifting electron density inhomogeneities (nonadiabatic electrostatic drift waves) and the well known adiabatic drift waves appears. The corresponding dispersion curves and amplification rates were determined for a slab model. The analytical results are illustrated by numerical calculations for the W VII-A stellarator plasma. An increase of more than an order of magnitude in comparison with previous theories was obtained for the amplification rate.

Introduction

The experiments show that the current driven drift waves could play an important role in the turbulent heat conductivity and in the diffusion of the plasma [5,8]. The theoretical amplification rates of the current driven drift instabilities are too small to be able to explain the experimental transport coefficients [1-4]. Therefore, we looked for an effect which could lead to an increase of the amplification rate. Such an effect is related to the following phenomenon:

The inhomogeneities of the electron and ion densities are drifting in an inhomogeneous plasma with the resulting velocity which corresponds to the diamagnetic and longitudinal electric currents. This motion of the inhomogeneities appears in the laboratory system as an electrostatic electron drift wave and an electrostatic ion drift wave. In both cases, the density changes are not adiabatic. Therefore, we call them nonadiabatic drift waves. In addition the usual adiabatic drift waves can propagate in the plasma slab. A special situation appears when the phase velocities and the wave vectors of the two waves (nonadiabatical and adiabatical) coincide. In these resonant conditions the amplification rate of one of the corresponding instabilities has a pronounced maximum.

In this paper we start from the two fluid theory, formulate the dispersion relation and calculate the amplification rates. All numerical applications concern the W VII-A stellarator plasma. At the end of the paper we give also the results about the variation of the wavelength of the most unstable current driven drift waves with the distance from the torus axis in the inside of the W VII-A stellarator plasma.

1) Parameters of the Equilibrium System

In the study of the drift wave, for simplicity, we consider $T_{\rm e} = T_{\rm i} = T$, and we assume that an inhomogeneous plasma exists in a uniform magnetic field. There are simultaneous density and

temperature gradients. We shall consider the case of a slab model, where the T = const and n = const surfaces are parallel planes. We choose the x-axis perpendicular to these surfaces in the direction of -grad n. In a quasineutral magnetohydrostatic equilibrium without electrical field $(n_e=n_i=n)$ the following relations

$$-\nabla p - \frac{ne}{c} \left[\vec{u}_e \times \vec{B} \right] = 0$$

$$-\nabla p + \frac{ne}{c} \left[\vec{u}_i \times \vec{B} \right] = 0$$

hold. Here we used the notations p = electron or ion pressure, n = electron or ion density, e = electric charge of ions, c = velocity of light, $\vec{B} = \text{magnetic}$ field intensity, $\vec{u}_e = \text{hydrodynamic}$ velocity of electrons, $\vec{u}_i = \text{hydrodynamic}$ velocity of ions. For the magnetohydrostatic equilibrium the following value results for the ion and electron velocities:

$$\vec{u}_e = -\vec{u}_i = -\frac{c}{enB} \frac{dp}{dx} \vec{n}_y = \vec{v}_d + \frac{c}{ds}$$
(1)

Here we assumed the y-axis in the direction of the electron drift. The unperturbed system is an inhomogeneous plasma with a diamagnetic current due to the relative drift of ions and electrons. We also assume that a longitudinal current (with respect to \vec{B})

$$\vec{\mathbf{z}}_{\mathbf{u}} = -\mathbf{ne}\,\vec{\mathbf{u}}$$

exists in the unperturbed system.

2) System of the Linearized Two Fluid Equations and the Dispersion Relation

If we neglect the electron inertia, the system of linearized two fluid equations for the small perturbations will have the following form:

$$n m_i \frac{d \vec{\vec{v}}_i}{dt} = -\nabla \vec{p}_i - ne \, \nabla \vec{\phi} + \frac{ne}{c} \left[\vec{\vec{v}}_i \times \vec{B} \right] - \frac{e}{c} \left[\vec{\vec{v}}_d \times \vec{B} \right] \vec{n}_i +$$

$$+\mu_{\perp}\nabla_{\perp}^{2}\vec{\vartheta}_{i} - \frac{ne}{\sigma_{o}}\vec{\mathring{d}} + \frac{ne\vec{\sigma}_{o}}{\sigma_{o}^{2}}\vec{\mathring{d}}_{\parallel}$$
 (3)

$$0 = -\nabla_{\parallel} \tilde{\rho}_{e} + ne \nabla_{\parallel} \tilde{\phi} + \frac{ne}{\sigma_{e}} \tilde{J}_{\parallel} - \frac{ne \tilde{\sigma}_{e}}{\sigma_{e}^{2}} \tilde{J}_{\parallel}$$
 (4)

$$0 = -\nabla_{\underline{I}} \tilde{p}_{e} + ne \nabla_{\underline{I}} \tilde{\phi} - \frac{ne}{c} [\tilde{\vec{v}}_{e} \times \vec{B}] - \frac{e}{c} [\tilde{\vec{v}}_{d} \times \vec{B}] \tilde{n}_{e}$$
 (5)

$$\frac{\partial \tilde{n}_{i}}{\partial t} + n \nabla_{\parallel} \tilde{v}_{i\parallel} + n \nabla_{\perp} \tilde{\vec{v}}_{i\perp} + \frac{dn}{dx} \tilde{v}_{ix} - v_{d} \frac{\partial \tilde{n}_{i}}{\partial y} = 0$$
 (6)

$$\frac{\partial \tilde{n}_{e}}{\partial t} + n \nabla_{\parallel} \tilde{\nabla}_{e\parallel} + n \nabla_{\perp} \tilde{\vec{v}}_{e\perp} + \frac{dn}{dx} \tilde{\nabla}_{ex} + v_{d} \frac{\partial \tilde{n}_{e}}{\partial y} + u \nabla_{\parallel} \tilde{n}_{e} = 0$$
 (7)

$$\frac{3}{2}n\left(\frac{\partial\widetilde{T}_{i}}{\partial t} + \frac{dT}{dx}\widetilde{v}_{ix} - v_{d}\frac{\partial\widetilde{T}_{i}}{\partial y}\right) + nT\left(\nabla_{\mu}\widetilde{v}_{i\mu} + \nabla_{\underline{L}}\widetilde{v}_{i\perp}\right) = 0$$
 (8)

$$\frac{3}{2}n\left(\frac{\partial \widetilde{T}_{e}}{\partial t} + \frac{dT}{dx}\widetilde{v}_{ex} + v_{d}\frac{\partial \widetilde{T}_{e}}{\partial y} + u\nabla_{\parallel}\widetilde{T}_{e}\right) + nT\left(\nabla_{\parallel}\widetilde{v}_{e\parallel} + \nabla_{\perp}\widetilde{v}_{e\perp}\right) = \alpha \frac{nT}{m_{e}v_{e}}\nabla_{\parallel}^{2}\widetilde{T}_{e}$$
(9)

$$\Delta \tilde{\phi} = -4\pi e (\tilde{n}_i - \tilde{n}_e) \tag{10}$$

$$\frac{\tilde{\vec{v}}}{\tilde{\vec{J}}} = en(\tilde{\vec{v}}_i - \tilde{\vec{v}}_e) - e\tilde{\vec{u}}\tilde{n}_e - e\tilde{\vec{v}}_d(\tilde{n}_e + \tilde{n}_i)$$
(11)

$$\tilde{\rho}_i = n \tilde{T}_i + T \tilde{n}_i$$
 (12)

$$\tilde{\rho}_{e} = n \tilde{T}_{e} + T \tilde{n}_{e}$$
 (13)

Here we have applied the usual notations. To denote the first order perturbations, we used the sign ~. The more important dissipative processes are considered, namely, the longitudinal electric resistivity, the transversal ion viscosity and the longitudinal thermal conductivity of the electrons. For the components parallel to the unperturbed magnetic field the subscript | and for the perpendicular components the subscript | has been used. The dissipative coefficients are

$$\mu_{\perp} = \frac{n \, \text{T} \, \nu_{ii}}{4 \, \Omega^2}; \quad \sigma_o = \frac{e^2 n}{m_e \nu_e}; \quad \alpha = 1,6 ; \quad (14)$$

 γ_{ii} is the ion-ion collision frequency, Ω = ion cyclotron frequency. In eqs. (3) and (4) the perturbation of the electrical conductivity appears. σ_{i} depends on temperature and on density (through the Coulomb logarithm). We have

$$\frac{\tilde{G}_{c}}{G_{c}} = \frac{3}{2} \frac{\tilde{T}_{e}}{T} + 8 \frac{\tilde{n}_{e}}{n} , \text{ with } 8 \approx \frac{1}{20}$$
 (15)

The system (3) - (13) is a complete closed system of equations. Supposing that the small perturbations are proportional to exp(-iwt + ikr), we get from (3) - (13) the following system of linear equations:

$$\begin{split} -i \left[k_{\parallel} - i \left(1 - \delta \right) \frac{\nu_{e} \xi}{\nu_{e}} \right] T \tilde{n}_{e} &- i \left(k_{\parallel} + \frac{3}{2} i \frac{\nu_{e} \xi}{\nu_{e}} \right) n \tilde{T}_{e} + \\ &+ i k_{\parallel} e n \tilde{\phi} + m_{e} \nu_{e} n \left(\tilde{\nu}_{i \parallel} - \tilde{\nu}_{e \parallel} \right) = 0 \\ - \frac{e B}{c} n \tilde{\nu}_{y e} - i \left(k_{x} - i \frac{e \nu_{d} B}{c T} \right) T \tilde{n}_{e} - i k_{x} n \tilde{T}_{e} + i k_{x} e n \tilde{\phi} &= 0 \\ \frac{e B}{c} n \tilde{\nu}_{x e} - i k_{y} T \tilde{n}_{e} - i k_{y} n \tilde{T}_{e} + i k_{y} e n \tilde{\phi} &= 0 \\ i k_{\parallel} T n \tilde{\nu}_{\parallel e} + i T \left(k_{x} - \frac{i}{n} \frac{d n}{d x} \right) n \tilde{\nu}_{x e} + i T k_{y} n \tilde{\nu}_{y e} - i \left(\omega - \omega_{p}^{*} - k_{\parallel} \omega \right) T \tilde{n}_{e} = 0 \end{split}$$

$$\begin{split} i\,k_{_{\parallel}}T_{n}\,\widetilde{\nu}_{_{\parallel e}}+i\,T\left(k_{_{x}}-i\,\frac{3}{2}\,\frac{1}{T}\frac{d\,T}{d\,x}\right)n\,\widetilde{\nu}_{_{x}e}+i\,T\,k_{_{y}}n\,\widetilde{\nu}_{_{y}e}\,-\\ &-\left[\frac{3}{2}\left(\omega-\omega_{_{P}}^{*}-k_{_{\parallel}}\omega\right)+i\,\frac{\alpha\,T}{m_{e}\nu}k_{_{\parallel}}^{2}\right]n\,\widetilde{T}_{e}=0 \end{split}$$

$$i\left[\left(\omega+\omega_{_{P}}^{*}\right)m_{_{i}}+i\,m_{e}\nu_{e}\right]n\,\widetilde{\nu}_{_{\parallel i}}+m_{e}\nu_{e}n\,\widetilde{\nu}_{_{\parallel e}}+\left(1-\delta\right)\frac{\nu_{e}\,\xi}{\nu_{e}}\,T\,\widetilde{n}_{e}\,-\\ &-\frac{3}{2}\,\frac{\nu_{e}\,\xi}{\nu_{e}}\,n\,\widetilde{T}_{e}-i\,k_{_{\parallel}}\,T\,\widetilde{n}_{_{i}}-i\,k_{_{\parallel}}\,n\,\widetilde{T}_{_{i}}-i\,k_{_{\parallel}}\,e\,\widetilde{n}\,\widetilde{\phi}\\ &-i\left[\left(\omega+\omega_{_{P}}^{*}\right)m_{_{i}}+i\left(\frac{\mu_{_{\perp}}\,k_{_{\perp}}^{2}}{n}\right]n\,\widetilde{\nu}_{_{x_{i}}}+\frac{e\,B}{c}\,n\,\widetilde{\nu}_{_{y_{i}}}-\\ &-i\left(k_{_{x}}-i\,\frac{e\,\nu_{e}\,B}{c\,T}\right)T\,\widetilde{n}_{_{i}}-i\,k_{_{x}}\,n\,\widetilde{T}_{_{i}}-i\,k_{_{x}}\,e\,\widetilde{n}\,\widetilde{\phi}\\ &=0 \end{split}$$

$$i\left[\left(\omega+\omega_{_{P}}^{*}\right)m_{_{i}}+i\left(\frac{\mu_{_{\perp}}\,k_{_{\perp}}^{2}}{n}\right)n\,\widetilde{\nu}_{_{y_{i}}}-\frac{e\,B}{c}\,n\,\widetilde{\nu}_{_{x_{i}}}-\\ &-i\,k_{_{y}}\,T\,\widetilde{n}_{_{i}}-i\,k_{_{y}}\,n\,\widetilde{T}_{_{i}}^{2}-i\,k_{_{y}}\,n\,\widetilde{\tau}_{_{i}}^{2}\\ &-i\,k_{_{y}}\,T\,\widetilde{n}_{_{i}}^{2}-i\,k_{_{y}}\,n\,\widetilde{T}_{_{i}}^{2}-i\,k_{_{y}}\,n\,\widetilde{\nu}_{_{y_{i}}}^{2}\\ &-i\,\frac{3}{2}\left(\omega+\omega_{_{P}}^{*}\right)T\,\widetilde{n}_{_{i}}^{2}+i\,k_{_{\parallel}}\,T\,n\,\widetilde{\nu}_{_{i\parallel}}+i\,T\left(k_{_{x}}-i\,\frac{3}{2}\,\frac{1}{T}\,\frac{d\,T}{d\,x}\right)n\,\widetilde{\nu}_{_{x_{i}}}^{2}+i\,k_{_{y}}\,T\,n\,\widetilde{\nu}_{_{y_{i}}}^{2}=0 \end{split}$$

$$-i\,\frac{3}{2}\left(\omega+\omega_{_{P}}^{*}\right)n\,\widetilde{T}_{_{i}}^{2}+i\,k_{_{\parallel}}\,T\,n\,\widetilde{\nu}_{_{i\parallel}}^{2}+i\,T\left(k_{_{x}}-i\,\frac{3}{2}\,\frac{1}{T}\,\frac{d\,T}{d\,x}\right)n\,\widetilde{\nu}_{_{x_{i}}}^{2}+i\,k_{_{y}}\,T\,n\,\widetilde{\nu}_{_{y_{i}}}^{2}=0 \end{split}$$

Here we have also used the notations $\xi = \frac{u}{v_e}$, $\omega_p = v_a k_y$ and $k_1 = k_x n_x + k_y n_y$. The system (16) has a nontrivial solution only if the determinant (17) vanishes:

Here
$$F = \frac{v_e}{v_e^2 k_{\parallel}^2}, \quad R = i \frac{19 v_e \xi}{20 v_e k_{\parallel}}, \quad S = i \frac{3}{2} \frac{\xi v_e}{v_e k_{\parallel}}, \quad Q = \frac{1.6 v_e^2 k_{\parallel}^2}{v_e},$$

$$G = i \frac{u_L c k_L^2}{n e B}, \quad P_e = \omega - \omega_p^* - u k_{\parallel}, \quad P_i = \omega + \omega_p^*, \quad J = \frac{v_i^2 k_x k_y}{\Omega_a},$$

$$\omega_n^* = -\frac{c k_y T}{e B n} \frac{dn}{dx}, \quad \omega_T^* = -\frac{c k_y}{e B} \frac{dT}{dx}, \quad \omega_p^* = v_d k_y = \omega_n^* + \omega_T^*,$$

$$\varepsilon = \frac{v_e^2 k_u^2}{\omega_{p\ell}^2} = \lambda_D^2 k^2 <<1$$

 \mathbf{v}_{i} = thermal velocity, \mathbf{v}_{e} = thermal velocity of electrons, $\pmb{\lambda}_{D}$ = Debye length.

The determinant (17) can be considered as the dispersion relation of the drift waves.

3) Classification of Possible Waves

As a first step to the interpretation of the dispersion relation we assume that the dissipative processes are negligable. Then from (15) the dispersion relation is

$$(\omega + \omega_{p}^{*})(\omega - \omega_{p}^{*} - u k_{\parallel}) \left[(9 + 30 g_{i}^{2} k_{x}^{2} + 15 i \frac{\omega_{n}^{*} k_{x}}{\Omega k_{y}})(\omega + \omega_{p}^{*})^{2} - (18 \omega_{T}^{*} + 33 \omega_{n}^{*})(\omega + \omega_{p}^{*}) + 3 (3 \omega_{T}^{*} - 2 \omega_{n}^{*})\omega_{p}^{*} - 30 v_{i}^{2} k_{\parallel}^{2} \right] = 0$$

(Q_i is the Larmor radius of the ions).

In deducing eq. (19) we have taken into account that we are looking for waves with frequency much less than the ion cyclotron frequency, such that $k_y \gg k_{\parallel}$ and $v_i k_y \gg \omega_p^*$. Eq. (19) has four different roots:

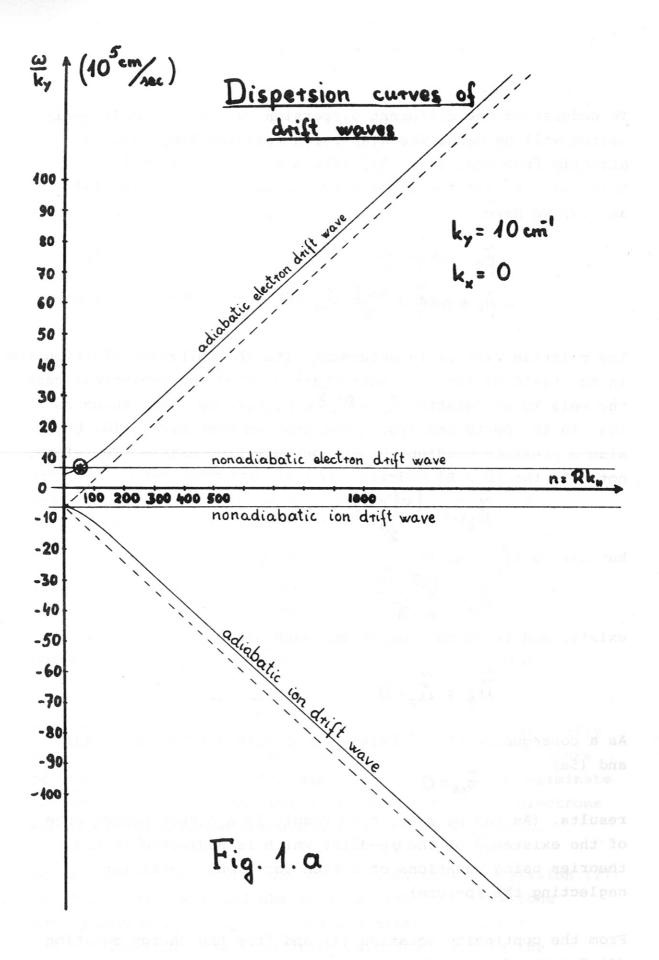
$$\omega = \omega_p^* + u k_{\parallel} = v_d k_y + u k_{\parallel}$$
 (20)

$$\omega = -\omega_p^* = -\nu_d k_y \tag{21}$$

and the two roots of the equation

$$f(\omega) = (9+30\varsigma_{i}^{2}k^{2}+15i\frac{\omega_{n}^{*}k_{x}}{\Omega k_{y}})(\omega+\omega_{p}^{*})^{2} - (18\omega_{T}^{*}+33\omega_{n}^{*})(\omega_{p}^{*}+\omega)+3(3\omega_{T}^{*}-2\omega_{n}^{*})\omega_{p}^{*}-30v_{i}^{2}k_{\parallel}^{2}=0$$

The corresponding dispersion curves are represented for $T_i = 167 \text{ eV}$, $\Omega = 3.10^8 \text{ sec}^{-1}$, $\frac{1}{2} \frac{\omega_y^2}{k_y} = \frac{\omega_x^2}{k_y} = \frac{\omega_y^2}{k_y} = \frac{3.22.10^5 \text{ cm/sec}}{k_y}$, u = 0 and $k_y = 10 \text{ cm}^{-1}$ in Fig. 1a.



To understand the different dispersion curves, a simple derivation will be made. The dispersion relation (20) results directly from eqs. (4), (5), (7), and (9). To demonstrate this, we consider for simplicity the case $k_{\rm X}=0$. From (4) and (5) we have

$$\tilde{\rho}_e - ne\tilde{\phi} = 0$$
 (4a)

$$-\tilde{p}_{e} + n\tilde{e}\tilde{\phi} + \frac{enB}{c}\tilde{v}_{xe} = 0$$
 (5a)

The relation (4a) is in agreement with the Boltzmann distribution in the field of the weak perturbation. (For the isothermal case the well known relation $\tilde{n}_e = \frac{\text{neo}}{T} \text{results}$). Eq. (4a) shows us that in the perturbed system not only an electric field, but also a pressure gradient of the perturbation exists. Therefore, not only the $\begin{bmatrix} \tilde{E} & x & \tilde{B} \end{bmatrix}$ - drift

$$\vec{\vec{u}}_{\phi} = -\frac{\left[\nabla \vec{\phi} \times \vec{\vec{B}}\right]}{B^2}$$

but also a $\nabla \tilde{p} - \text{drift}$ $\tilde{\vec{u}}_p = \frac{\left[\nabla \tilde{p} \times \tilde{B}\right]}{\text{ne } B^2}$

exists, and these two compensate each other.

As a consequence of the existence of both drifts, from (4a) and (5a) $\overset{\sim}{v_{xe}} = 0$

results. (As can be seen, this result is a direct consequence of the existence of the ∇p -drift which is neglected in the theories using equations of motion for simple particles neglecting the ∇p -term).

From the continuity equation (7) and from the energy equation (9) for the dissipation free system we get

$$-i(\omega - k_y v_d - k_{\parallel} u) \tilde{n}_e + in(k_{\parallel} \tilde{v}_{e\parallel} + k_y \tilde{v}_{ey}) + \frac{dn}{dx} \tilde{v}_{ex} = 0$$

$$-i(\omega - k_y v_d - k_{\parallel} u) \frac{3}{2} \tilde{n}_e + inT(k_{\parallel} \tilde{v}_{e\parallel} + k_y \tilde{v}_{ey}) + \frac{dT}{dx} \tilde{v}_{ex} = 0$$

Taking into account that $v_{ex} = 0$, we have

$$\left(\omega - k_y v_d - k_{\parallel} u\right) \left(T \tilde{n}_e - \frac{3}{2} n \tilde{T}_e\right) = 0 \tag{23}$$

This equation shows that we have two kinds of perturbations:

a) adiabatic if

$$T\tilde{n}_e - \frac{3}{2}n\tilde{T}_e = 0$$
 , and

b) nonadiabatic if

For the <u>nonadiabatic</u> case we got exactly the dispersion relation (20). In this case, we have a nonadiabatic accumulation of electrons in some regions of the plasma. The magnetic field prevents the neutralization of the charges and leads to a drift motion of the charged regions with electron surplus with the resulting macroscopical drift velocity

This movement appears in the laboratory system as an electrostatic wave with the dispersion relation (20).

For the adiabatic motion, we got the well known relation

$$\tilde{n}_e = \frac{3}{2} \frac{n}{T} \tilde{T}_e$$

Therefore, if we use this supposition about the adiabaticity (or another supposition about an eventual isothermal nature of the wave) instead of the energy relation (9), we eliminate automatically the nonadiabatic accumulation of the electrons and the corresponding nonadiabatic electrostatic wave.

We can show in a similar way that the dispersion relation (21) is related to the nonadiabatic accumulation of the ions moving macroscopically with the ion drift velocity - and leading therefore to a drift wave described in the laboratory system by (21).

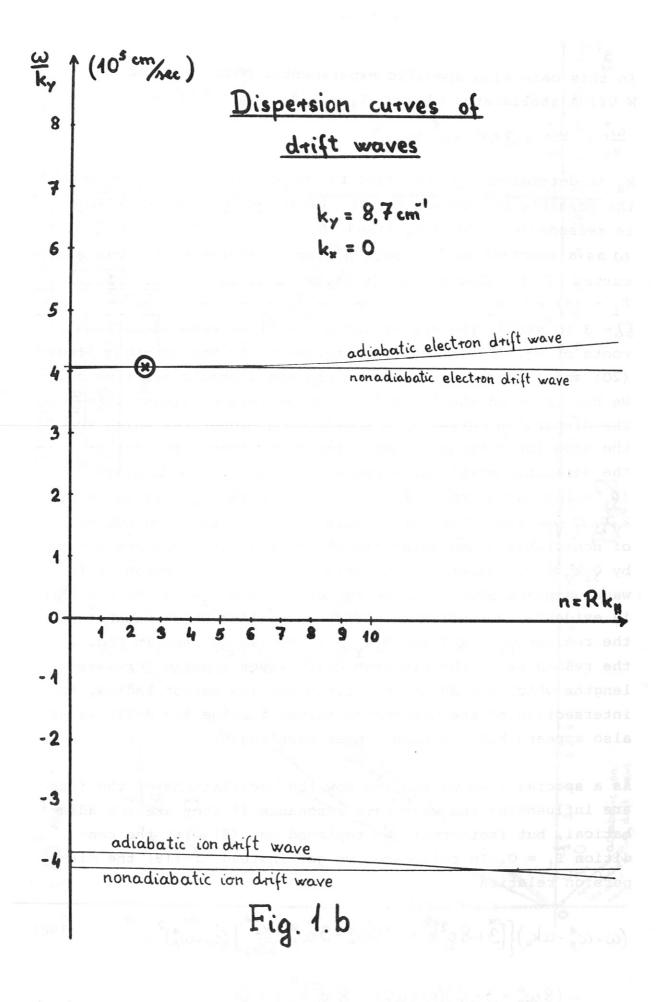
Now we can return to Fig. 1a). To have s similarity with the well known dispersion curves [1] we plotted ω/k_{v} as a function of k_{\parallel} $(\frac{n}{200})$ for a fixed k_{y} value. Here we did not use the system moving with the ions, but the laboratory system in which the ions have the velocity - Vaty . Therefore, the origin of our coordinate system is shifted in comparison with the system used in [1]. As can be seen, we get the typical dispersion curves of the two adiabatic drift waves, but some other two dispersion curves appear which are related to the nonadiabatic accumulation of the drifting particles. These dispersion curves are intersecting each other, and therefore, the existence of a resonance phenomenon is expected to appear which is characterized by the coincidence of the frequencies and wave vectors of the two resonating waves. The importance of this resonance results from the following observation:

As a consequence of the dissipation the rigt hand side of (19) is different from zero; it will be written in the form i δ . (The explicit expression of δ will be given later). In this case the dispersion relation (19) is written in the form

$$\omega - \omega_{p}^{*} - u k_{\parallel} = \frac{i\delta}{(\omega + \omega_{p}^{*}) f(\omega)}$$
(24)

The expression on the right-hand side is only a small correction as long as the root of (20) does not coincide with the root of (22). In the vicinity of the intersections of the dispersion curves (20) and (22) the right-hand side of (23) becomes very large and we get a very great amplification (or absorption) rate of the waves. Therefore, the intersection points of the dispersion curves give a first indication on the wavelength of the most unstable perturbations.

In Fig. 1a) the intersection point lies at very high toroidal harmonics (n \approx 50). The position of the intersection point strongly depends on k. In Fig. 1b) we represented a case (k_y = 8.7 cm⁻¹) which seems to be more important in the experimental situations. Here the intersection point is in the vicinity of the second and third toroidal harmonics. (We used



in this case also specific experimental parameters of the W VII-A stellarator plasma, $T_i = 129$ eV, $\Omega = 3.40^8 \text{ sec}^4$,

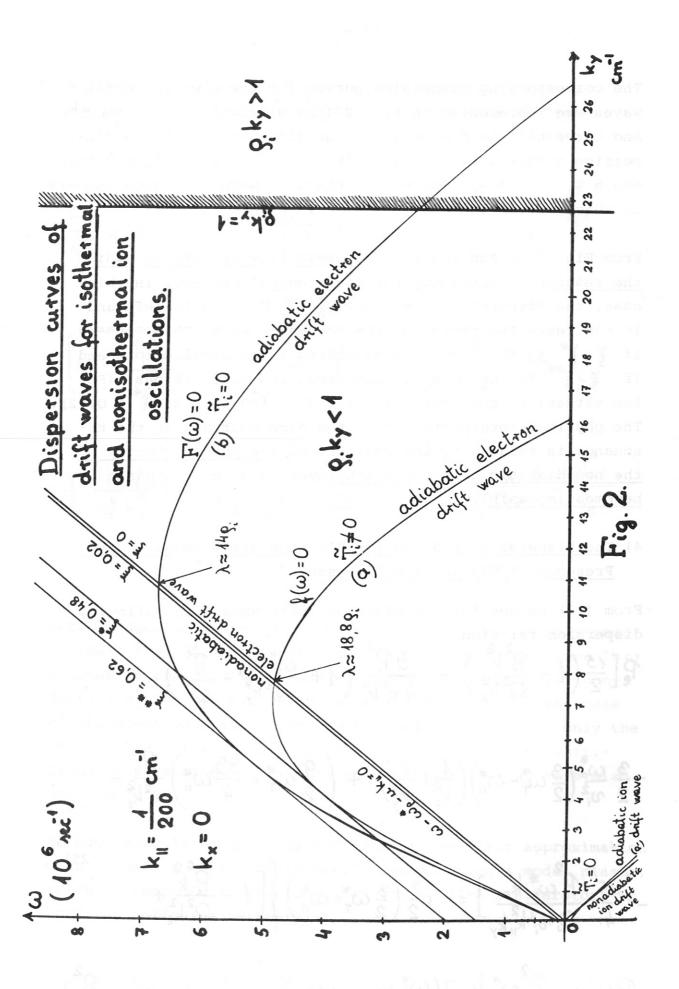
$$\frac{\omega_{T}^{*}}{k_{y}} = \frac{\omega_{n}^{*}}{k_{y}} = 2,05.10^{5} \text{ cm/sec}$$
).

 k_{\parallel} is determined by the major radius of the torus. To determine the possible resonances for the first toroidal harmonics, it is reasonable to hold k_{\parallel} fixed $(k_{\parallel} = \frac{1}{R} = \frac{1}{200} \text{ cm}^{-1})$ and to plot $\boldsymbol{\omega}$ as a function of $\boldsymbol{k}_{\boldsymbol{V}}.$ Such a representation of the dispersion curves of the electron drift waves is given in Fig. 2 for $T_{i} = 167 \text{ eV}, \ v_{d} = 6.44 \cdot 10^{5} \text{ cm sec}^{-1}, \ k_{II} = 0.5 \cdot 10^{-2} \text{cm}^{-1}$ and Ω = 3·10⁸sec⁻¹. The curves (a) and (a') correspond to the roots of eq. (22). They are intersected by the straight lines (20) and (21) corresponding to the nonadiabatic drift waves. We have plotted the lines (20) for different values of $\xi = \frac{u}{v_e}$. The dispersion curves have some common properties which are the same for various plasmas. The dispersion curve (a) of the adiabatic drift waves has a maximum for $k_{v}Q_{i}\approx 0.318$ $(\mathbf{Q}_{i} = \text{ion Larmor radius})$ and intersects the \mathbf{k}_{y}^{-} -axis in point $k_{v} Q_{i} \approx \frac{1}{\sqrt{2}}$. The intersection point of the dispersion curves of nonadiabatic and adiabatic electron drift waves are always by $q_i k_v \leqslant \frac{1}{3}$. Therefore, the resonance of the electron drift waves appears always in the region where $Q_i k_y <$ 1. To put this in evidence, we indicate in Fig. 2 the line which separates the regions $q_{i}k_{y}$ < 1 and $q_{i}k_{y}$ > 1. As can be seen in Fig. 2 the resonance of the electron drift waves appears for wavelengths which are about 18.8 times the ion Larmor radius. An intersection of the dispersion curves for the ion drift waves also appears but for much longer wavelength.

As a special case we studied how the oscillations of the ions are influencing the wave-wave resonance if they are not adiabatical, but isothermal. We replaced eq. (8) with the condition $\tilde{T}_i = 0$. In this case, we get instead of (19) the dispersion relation

$$(\omega - \omega_{p}^{*} - \mu k_{\parallel}) \left\{ \left[3 + 8 \varsigma_{i}^{2} k^{2} + i \left(8 \omega_{n}^{*} - 3 \omega_{p}^{*} \right) \frac{k_{x}}{\Omega k_{y}} \right] \left(\omega + \omega_{p}^{*} \right)^{2} - \left(8 \omega_{n}^{*} + 3 \omega_{p}^{*} \right) \left(\omega + \omega_{p}^{*} \right) - 8 \upsilon_{i}^{2} k_{\parallel}^{2} \right\} = 0$$

$$(19a)$$



The corresponding dispersion curves for the electron drift waves are represented in Fig. 2 (the straight line $\omega \cdot \omega_i^* - u k_{\parallel}$ and curve(b)). As can be seen, the intersection of the dispersion curves appears again. It is shifted to smaller λ from $\sim 18.8 \ Q_i$ to $\sim 14 \ Q_i$. Therefore, the wave-wave resonance appears in this case, too.

From Fig. 2 we can see that a possibility appears to avoid the resonance increasing the longitudinal current. In this case, the dispersion curve $\omega - \omega_{\rho}^{2} - \mu k_{\parallel} = 0$ moves to left and it can leave the region of the adiabatic wave. This appears if λ_{ρ}^{2} in the case of nonisothermal ion oscillations and if λ_{ρ}^{2} in the case of isothermal ion oscillations. (For the situation represented in Fig. 2 $\lambda_{\rho}^{2} = 0.48$ and $\lambda_{\rho}^{2} = 0.62$). The physical interpretation of the disappearance of the resonance is related to the increase of the phase velocity of the nonadiabatic electron drift waves, and the resonance becomes impossible.

4) The Dispersion Relation for Electron Drift Waves in the Presence of Dissipative Processes

From (17) we get for the electron drift waves the following dispersion relations:

$$P_{e}\left[\frac{15}{2}\left(1 - \frac{P_{i}^{2}k_{u}^{2}}{Q^{2}k_{u}^{2}}\right) - \frac{9P_{i}^{2}}{4v_{i}^{2}k_{u}^{2}}\left(1 + \frac{\omega_{p}^{*}\omega_{n}^{*}}{v_{i}^{2}k_{y}^{2}} - \frac{P_{i}^{2}}{Q^{2}}\right) - \frac{9P_{i}^{2}}{4v_{i}^{2}k_{u}^{2}}\left(1 + \frac{\omega_{p}^{*}\omega_{n}^{*}}{v_{i}^{2}k_{y}^{2}} - \frac{P_{i}^{2}}{Q^{2}}\right)\right)$$

$$-\frac{3}{2} \frac{\omega_{r}^{*}}{v_{i}^{2}} \left(\frac{3}{2} \omega_{r}^{*} - \omega_{n}^{*}\right) \left(\frac{1}{k_{u}^{2}} + \frac{1}{k_{y}^{2}}\right) \\ + \left(\frac{9}{2} \omega_{r}^{*} + \frac{33}{4} \omega_{n}^{*}\right) \frac{P_{i}}{v_{i}^{2} k_{u}^{2}} -$$

$$-i\frac{15}{4}\frac{P_{i}^{2}\omega_{n}^{*}k_{x}}{\Omega_{i}v_{i}^{2}k_{\parallel}^{2}k_{y}}=i\frac{3}{2}\left(\frac{3}{2}\omega_{+}^{*}-\omega_{n}^{*}\right)\left\{\left[1-\frac{P_{i}^{2}k_{\perp}^{2}}{\Omega_{i}^{2}k_{\parallel}^{2}}+\right]\right\}$$

$$+ \frac{P_{i}\omega_{n}^{*}}{v_{i}^{2}k_{II}^{2}} - i \frac{P_{i}^{2}\omega_{n}^{*}k_{x}}{\Omega v_{i}^{2}k_{II}^{2}k_{y}} \Big] \Big(\frac{\omega_{p}^{*}\nu_{e}}{v_{e}^{2}k_{II}^{2}} + \frac{49\nu_{e}\xi}{20\nu_{e}k_{II}} \Big) - \Big[\omega_{p}^{*} - P_{i}\Big(1 - \frac{P_{i}^{2}}{\Omega^{2}}\Big) - \frac{1}{20\nu_{e}k_{II}} \Big] \Big(\frac{2\nu_{p}^{*}}{\nu_{e}^{2}k_{II}^{2}} + \frac{29\nu_{e}\xi}{20\nu_{e}k_{II}} \Big) - \frac{1}{20\nu_{e}k_{II}} \Big(\frac{2\nu_{p}^{*}}{20\nu_{e}k_{II}} + \frac{29\nu_{e}\xi}{20\nu_{e}k_{II}} + \frac{29\nu_{e}\xi}{20\nu_{e}k_{II}} \Big) - \frac{1}{20\nu_{e}k_{II}} \Big(\frac{2\nu_{p}^{*}}{20\nu_{e}k_{II}} + \frac{29\nu_{e}\xi}{20\nu_{e}k_{II}} + \frac{29\nu_{e}\xi}{20\nu_{e}k_{II}} \Big) - \frac{1}{20\nu_{e}k_{II}} \Big(\frac{2\nu_{p}^{*}}{20\nu_{e}k_{II}} + \frac{29\nu_{e}\xi}{20\nu_{e}k_{II}} \Big) - \frac{1}{20\nu_{e}k_{II}} \Big(\frac{2\nu_{p}^{*}}{20\nu_{e}k_{II}} + \frac{29\nu_{e}\xi}{20\nu_{e}k_{II}} + \frac{29\nu_{e}\xi}{20\nu_{e}k_{II}} \Big) - \frac{1}{20\nu_{e}k_{II}} \Big(\frac{2\nu_{e}^{*}}{20\nu_{e}k_{II}} + \frac{2\nu_{e}^{*}}{20\nu_{e}k_{II}} \Big) - \frac{2\nu_{e}^{*}}{20\nu_{e}k_{II}} \Big(\frac{2\nu_{e$$

$$-\frac{P_{i}\,\omega_{p}^{*}\,\omega_{n}^{*}}{v_{i}^{2}\,k_{y}^{2}}+i\,\frac{P_{i}\,\omega_{p}^{*}\,k_{x}}{\Omega\,k_{y}}\bigg]\frac{\nu_{e}}{v_{e}^{2}\,k_{n}^{2}}+\bigg[\frac{5}{2}\Big(1-\frac{P_{i}^{2}k_{x}^{2}}{\Omega^{2}k_{n}^{2}}\Big)+\frac{3P_{i}\,\omega_{p}^{*}}{v_{i}^{2}\,k_{n}^{2}}-$$

$$-\frac{3 P_{i}^{2}}{2 v_{i}^{2} k_{\parallel}^{2}} \left(1-\frac{P_{i}^{2}}{\Omega^{2}}\right)-\frac{3 P_{i}^{2} \omega_{p}^{*} \omega_{n}^{*}}{2 v_{i}^{4} k_{\parallel}^{2} k_{y}^{2}}-\left(\frac{3}{2} \omega_{T}^{*}-\omega_{n}^{*}\right) \frac{\omega_{p}^{*}}{v_{i}^{2}} \left(\frac{1}{k_{\parallel}^{2}}+\frac{1}{k_{y}^{2}}\right)\right] \frac{v_{e} \xi}{v_{e} k_{\parallel}}\right\}-$$

$$-i \left[4 \left(1 - \frac{{{\mathcal{P}_{i}}^{2} {\mathsf{k}_{ii}^{2}}}}{{{\Omega_{i}}^{2} {\mathsf{k}_{ii}^{2}}}} \right) - \left(\frac{3}{2} \, \omega_{\mathsf{T}}^{*} - \omega_{\mathsf{n}}^{*} \right) \frac{{\omega_{\mathsf{p}}^{*}}}{v_{i}^{2}} \left(\frac{1}{{\mathsf{k}_{\mathsf{H}}^{2}}} + \frac{1}{{\mathsf{k}_{\mathsf{y}}^{2}}} \right) + \frac{{{\mathcal{P}_{i}}}}{{\mathsf{k}_{\mathsf{H}}^{2}} v_{i}^{2}} \left(3 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) - \frac{1}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) - \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{n}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{p}}^{*} \right) + \frac{2}{2} \left(2 \, \omega_{\mathsf{p}}^{*} + \frac{3}{2} \, \omega_{\mathsf{p}}^$$

$$-\frac{3}{2} \frac{P_{i}^{2} \omega_{p}^{*} \omega_{n}^{*}}{v_{i}^{4} k_{\parallel}^{2} k_{y}^{2}} - \frac{3}{2} \frac{P_{i}^{2}}{k_{\parallel}^{2} v_{i}^{2}} \left(1 - \frac{P_{i}^{2}}{\Omega^{2}}\right) - i \frac{3}{2} \frac{P_{i}^{2} \omega_{n}^{*} k_{x}}{v_{i}^{2} k_{\parallel}^{2} \Omega k_{y}} \frac{1.6 v_{e}^{2} k_{\parallel}^{2}}{\nu_{e}}$$

Here we used the same notation as in (17). The imaginary part on the r.h.s. of the dispersion relation which determines ${\rm Im}\,\omega$ depends on the electric conductivity and on the thermal conductivity. The ion viscosity is of no importance in the case of the electron drift waves. In the deduction of (25) only the term of the first order with respect to dissipative coefficients was retained. The terms proportional to $k^2\lambda_D^2$ were also neglected.

We have calculated $V = \text{Im} \omega$ from (25) in the first approximation with respect to dissipative coefficients, using a model plasma, which corresponds to the plasma of the W VII-A stellarator. We used the following parameters:

$$\omega_{p}^{*} = 2\omega_{T}^{*} = 2\omega_{n}^{*} = 3.10^{5} \, \text{ky} \text{ (sect)}$$
 $v_{i} \approx 1.5 \cdot 10^{7} \, \text{cm sect}$
 $v_{e} \approx 6.10^{8} \, \text{cm sect}$
 $v_{e} \approx 6.10^{8} \, \text{cm sect}$
 $v_{e} \approx 6.10^{8} \, \text{cm sect}$
 $v_{e} \approx 3.10^{7} \, \text{cm} \text{ sect}$
 $v_{e} \approx 3.10^{8} \, \text{cm}^{2}$
 $v_{e} \approx 3.10^{8} \, \text{sect}$

The more important dissipative effect in (25) is the longitudinal electric conductivity. Therefore, we looked for the solution of eq. (25) neglecting the last term on the r.h.s. describing the influence of the longitudinal thermal conductivity of the electrons. The dispersion relation (25) can be written in this case in the form

$$A P_{e}^{3} + (B-iC) P_{e}^{2} + (D-iE) P_{e} - iF = 0$$
 (27)

The coefficients of this equation are:

The coefficients of this equation are:
$$A = \frac{15}{2} \frac{k_{y}^{2}}{\Omega^{2} k_{\parallel}^{2}} + \frac{9}{4 v_{i}^{2} k_{\parallel}^{2}}$$

$$B = \frac{60 v_{n}}{\Omega^{2} k_{\parallel}^{2}} k_{y}^{3} + \frac{15 u}{\Omega^{2} k_{\parallel}} k_{y}^{2} + \frac{21}{4} \frac{v_{n}}{v_{i}^{2} k_{\parallel}^{2}} k_{y} + \frac{9 u}{2 v_{i}^{2} k_{\parallel}}$$

$$C = \frac{3}{2} \frac{v_{n}^{2} v_{e}}{\Omega^{2} v_{e}^{2} k_{\parallel}^{4}} k_{y}^{4} + 3.7125 \frac{v_{n} v_{e} u}{\Omega^{2} v_{e}^{2} k_{\parallel}^{3}} k_{y}^{3} + \frac{9 v_{n} u v_{e}}{8 v_{i}^{2} v_{e}^{2} k_{\parallel}^{3}} k_{y}$$

$$D = \frac{120 v_{n}^{2} k_{\parallel}^{4}}{\Omega^{2} k_{\parallel}^{2}} k_{y}^{4} + \frac{60 v_{n} u}{\Omega^{2} k_{\parallel}} k_{y}^{3} - \frac{27}{2} \frac{v_{n}^{2} k_{y}^{2}}{v_{i}^{2} k_{\parallel}^{2}} +$$

$$(28)$$

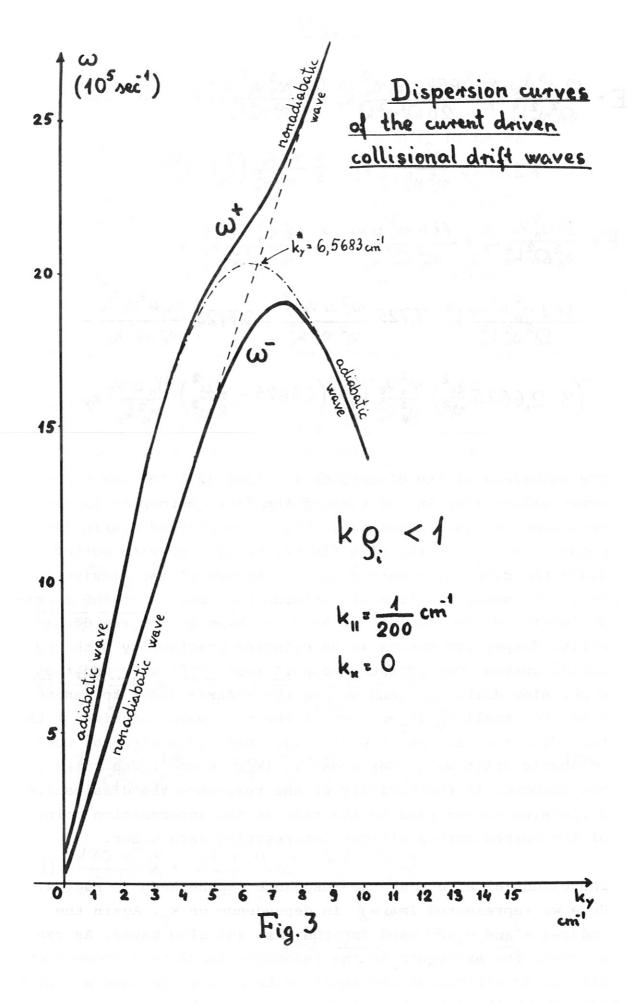
 $+\frac{21}{\mu}\frac{v_nu}{v_1^2k}k_y-\frac{15}{2}+\frac{9}{\mu}\frac{u^2}{v_1^2}$

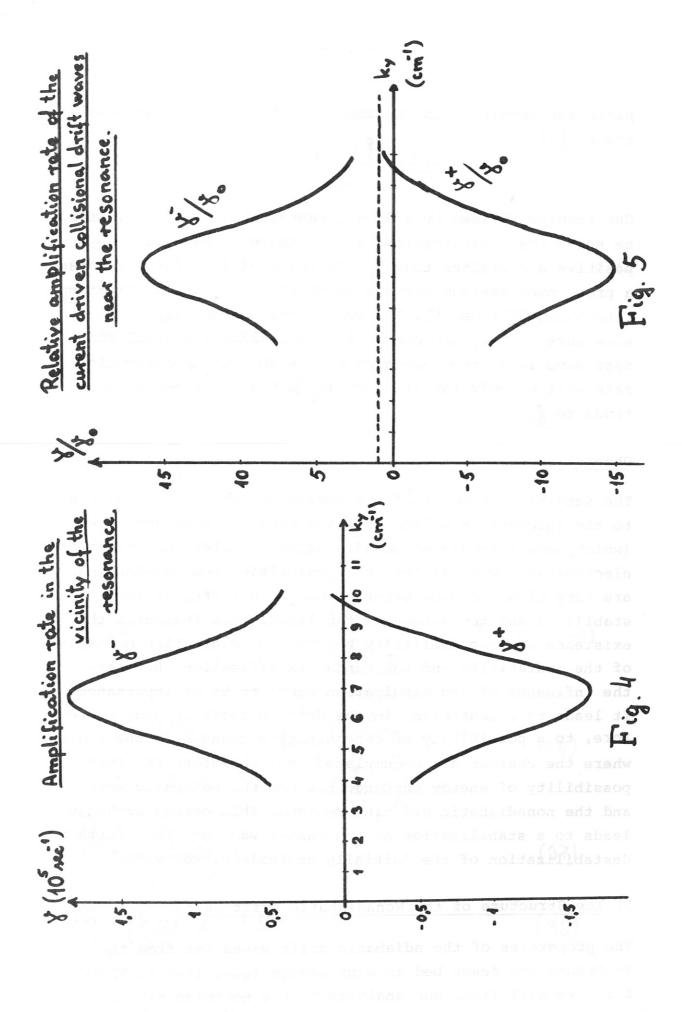
$$\begin{split} E &= \frac{12 \, v_{n}^{3} \, v_{e}}{\Omega^{2} \, v_{e}^{2} \, k_{u}^{4}} \, k_{y}^{5} + \frac{654 \, u \, v_{e} \, v_{n}^{2}}{20 \, \Omega^{2} \, v_{e}^{2} \, k_{u}^{3}} \, k_{y}^{4} - \frac{3}{2} \, \frac{v_{n}^{3} \, v_{e}}{v_{i}^{2} \, v_{e}^{2} \, k_{u}^{4}} \, k_{y}^{3} + \\ &\quad + 2.6625 \, \frac{v_{n}^{2} \, u \, v_{e}}{v_{i}^{2} \, v_{e}^{2} \, k_{u}^{3}} \, k_{y}^{2} + \frac{3}{4} \, \frac{v_{n} \, v_{e}}{v_{e}^{2} \, k_{u}^{2}} \left(\frac{3u^{2}}{v_{i}^{2}} - 1 \right) k_{y} \end{split}$$

$$F &= \frac{24 \, v_{n}^{4} \, v_{e}}{v_{e}^{2} \, \Omega^{2} \, k_{u}^{4}} \, k_{y}^{6} + \frac{71.4 \, v_{n}^{3} \, u \, v_{e}}{v_{e}^{2} \, \Omega^{2} \, k_{u}^{3}} \, k_{y}^{5} - \frac{6 \, v_{n}^{4} \, v_{e}}{v_{i}^{2} \, v_{e}^{2} \, k_{u}^{4}} \, k_{y}^{4} + \\ &\quad + \frac{31.2 \, v_{n}^{2} \, u^{2} \, v_{e}}{\Omega^{2} \, k_{u}^{2}} \, k_{y}^{4} - 7.725 \, \frac{v_{n}^{3} \, u \, v_{e} \, k_{y}^{3}}{v_{i}^{2} \, v_{e}^{2} \, k_{u}^{3}} + 3.7125 \, \frac{v_{n} \, u^{3} \, v_{e} \, k_{y}^{4}}{v_{e}^{2} \, \Omega^{2} \, k_{u}^{4}} - \\ &\quad - \left(3 - 2.6625 \, \frac{u^{2}}{v_{i}^{2}} \right) \, \frac{v_{n}^{2} \, v_{e}}{v_{e}^{2} \, k_{u}^{2}} \, k_{y}^{2} - \left(4.4625 - \frac{9 \, u^{2}}{8 \, v_{i}^{2}} \right) \, \frac{v_{n} \, u \, v_{e}}{v_{e}^{2} \, k_{u}^{4}} \, k_{y}^{2} \end{split}$$

The solutions of the dispersion relation (27) for the parameter values (26) in the case of the first harmonics is represented in Figs. 3 and 4. In Fig. 3 we plotted Rewin dependence on $\boldsymbol{k}_{_{\boldsymbol{V}}}.$ In the same figure, we plotted with dotted lines the dispersion curves in the absence of the dissipation. As can be seen, far from the intersection points of the dissipationfree dotted curves, we have two branches of the dissipative dispersion curves which coincide practically with the dotted curves. One of these we call fast drift wave and the other slow drift wave and we use the index + and - to denote them. For small $k_{_{\rm V}}$ ($k_{_{\rm V}}<$ 5 cm $^{-1}$) the fast wave coincides with the adiabatic wave and the slow wave coincides with the nonadiabatic drift wave. For great k_y ($k_y > 8$ cm⁻¹) the roles are changed. In the vicinity of the resonance the dissipative dispersion curves pass by the side of the intersection point of the dotted curves without intersecting each other.

The essential effect of the resonance can be seen in Fig. 4. Here we represented ${\rm Im}\,\omega=\zeta$ in dependence on $k_{\rm y}$. Again the indices + and - are used for the fast and slow waves. As can be seen, the existence of the resonance leads to a pronounced maximum or minimum of the amplification rate. To have a better idea about the influence of the resonance phenomenon, we com-





pared our results with the amplification rate of the usual theory [2] y = 0.51 ξνεωρ k.νο

(29)

Our results divided by (29) are represented in Fig. 5. As can be seen, the amplification rate of the slow wave is always positive and greater than χ_0 . In the vicinity of the resonance a pronounced maximum appears which leads to an amplification rate about 16 times χ_0 . Therefore, the instability of the slow wave is very pronounced in the resonance region. The fast wave is at the resonance stable and the amplification rate will be only for great k_y ($k_y \gtrsim 9.5 \text{ cm}^{-1}$) positive and tends to X.

Observation

The sensitive increase of the amplification rate is related to the interaction of the drifting density inhomogeneities (which were accumulated nonadiabatically) with the adiabatic electrostatic wave if the phase velocities and wavelengths are very close to each other. Always the faster of them is stabilized and the slower destabilized. This indicates the existence of some similarity between the excitation mechanism of the instability and the Landau amplification. But here the influence of the dissipation seems to be of importance. It leads to a scattering in the drift velocities and, therefore, to a possibility of concentration changes in the regions where the charges are accumulated, and therefore it gives the possibility of energy exchange between the adiabatic waves and the nonadiabatic drifting regions. This energy exchange leads to a stabilization of the faster wave and to a further destabilization of the initially unstable slower wave.

5) The Structure of the Nonadiabatic Drift Waves

The properties of the adiabatic drift waves far from the resonance are described in many papers (e.g. [1-4]). Therefore, we will limit our analysis to the nonadiabatic drift waves (only to the case, if the resonance has no essential

influence). In this case, the dissipative process has no essential influence on the motion of the particles.

a) Neglecting the dissipation, we get the following expression for the dependence of the small perturbations from the density perturbation (\tilde{n}_e) in the case of nonadiabatic electron drift wave.

wave.

$$\tilde{n}_{i} = \tilde{n}_{e} + O(k^{2}\lambda_{D}^{2}) \qquad (30)$$

$$\tilde{v}_{ell} = \frac{\omega_{p}^{*}}{k_{ll}} \frac{\tilde{n}_{e}}{n} \qquad (31)$$

$$\tilde{v}_{ex} = 0 \qquad (32)$$

$$\tilde{v}_{ey} = -v_{d} \frac{\tilde{n}_{e}}{n} \qquad (33)$$

$$v_{ill} = -\frac{\left(P - \omega_{p}^{*} - i \frac{P \omega_{p}^{*} k_{x}}{\Omega k_{y}}\right) v_{i}^{2} k_{ll}}{P \omega_{n}^{*} + v_{i}^{2} k_{ll}^{2} - i \frac{P^{2} \omega_{n}^{*} k_{x}}{k_{y}^{2}} \left(1 + \frac{P \omega_{p}^{*}}{V_{i}^{2} k_{ll}^{2}}\right)\right] v_{i}^{2} k_{y}} \frac{n_{e}}{n} \qquad (35)$$

$$v_{iy} = -i \frac{\left(\omega_{p}^{*} - P + i \frac{P^{2} k_{x}}{\Omega k_{y}} + v_{i}^{2} k_{ll}^{2} - i \frac{P^{2} \omega_{n}^{*} k_{x}}{\Omega k_{y}}\right) p_{i} v_{i}^{2} k_{y}}{P \omega_{n}^{*} + v_{i}^{2} k_{ll}^{2} - i \frac{P^{2} \omega_{n}^{*} k_{x}}{\Omega k_{y}}} \frac{\tilde{n}_{e}}{n} \qquad (36)$$

$$\tilde{\Phi} = \frac{2T\Delta_{1}}{3ne\Delta} \tilde{n}_{e} \qquad (37)$$

$$\tilde{T}_{e} = \left(\frac{2\Delta_{1}}{3\Delta_{1}} - 1\right) \frac{T}{n}e \qquad (39)$$

where
$$\Delta_{1} = \frac{3}{2} p^{2} - \frac{5}{2} v_{i}^{2} k_{\parallel}^{2} \left(1 - \frac{p^{2} k^{2}}{\Omega^{2} k_{\parallel}^{2}}\right) - 3P \omega_{p}^{*} + \left(\frac{3}{2} \omega_{\tau}^{*} - \omega_{n}^{*}\right) \omega_{p}^{*}, \quad (40)$$

$$\Delta_{2} = \frac{3}{2} P \omega_{T}^{*} + v_{i}^{2} k_{\parallel}^{2} \left(1 - \frac{P^{2} k_{\parallel}^{2}}{\Omega^{2} k_{\parallel}^{2}}\right) - \left(\frac{3}{2} \omega_{T}^{*} - \omega_{n}^{*}\right) \omega_{p}^{*} - i \frac{3}{2} \frac{P^{2} \omega_{T}^{*} k_{x}}{\Omega k_{y}}, \quad (41)$$

$$\Delta = P \omega_n^* + v_i^2 k_{||}^2 - i \frac{P^2 \omega_n^* k_x}{\Omega k_y}, \qquad (42)$$

and

$$P = 2\omega_p^* + \mu k_{\parallel} \tag{43}$$

Here we have used the same approximation as in the deduction of eq. (19). Relation (30) shows that the deviation from electroneutrality is of the order of $k^2 \lambda_D^2$. From (32) it results that the electron oscillations are localized to the surfaces p = const. These surfaces are deformed by ion oscillations only ($\tilde{v}_{ix} \neq 0$). From (31) - (33) the transversality relation

$$kv = 0$$

for the electron oscillations follows. In the case of the ions, the longitudinal oscillations (with respect to k) do not cancel. The finite inertia of the ions leads to the deformation of the surfaces p=const. For $k_x=0$ we get

$$\frac{\tilde{v}_{ix}}{\tilde{v}_{i||}} = -i \frac{\rho k_y}{\Omega k_{||}}$$
(44)

The phase difference of $\frac{\pi}{2}$ between \tilde{v}_{i*} and \tilde{v}_{i*} indicates the convective motion of the particles, which can lead to the appearance of a convective instability. For the first toroidal harmonics in the vicinity of the resonance $|\tilde{v}_{i*}| \approx |\tilde{v}_{i*}|$

b) Ion Drift Waves

For the relations between the perturbations of an ion drift wave in the absence of dissipative effects, we obtain from (16)

$$\tilde{n}_e \approx \tilde{n}_i$$
 (45)

$$\tilde{v}_{ij} = -\frac{\omega_p^*}{k_y} \frac{\tilde{n}_i}{n} \tag{46}$$

$$\tilde{v}_{iy} = \frac{\omega_p^*}{k_y} \frac{\tilde{n}_i}{n}$$
(47)

$$\tilde{v}_{ix} = 0$$
 (48)

$$\tilde{N}_{ell} = -\frac{\omega_p^* + u \, k_{||}}{k_{||}} \, \frac{\tilde{n}_i}{n} \tag{49}$$

$$\tilde{v}_{ey} = -\frac{\omega_p^*}{k_y} \frac{\tilde{n}_i}{n}$$
 (50)

$$\tilde{v}_{ex} = 0 \tag{51}$$

$$\frac{\tilde{T}_{e}}{T} \approx \frac{2}{3} \frac{\tilde{n}_{e}}{n} \tag{52}$$

$$\frac{\widetilde{T}_{i}}{T} \approx \frac{8}{3} \frac{\widetilde{n}_{i}}{n}$$
 (53)

$$\tilde{J}_{\parallel} = -e u \tilde{n} e$$
 (54)

$$\tilde{\phi} \approx \frac{5}{3} \frac{T}{ne} \tilde{n}_e \tag{55}$$

Hence the deviation from quasineutrality is again negligible. Both the ion oscillations and the electron oscillations are localized to the p = const planes. There are no oscillations which would modify the planes p = const. From (46), (47) the transversality relation $kv_1 = 0$ for the ion oscillations results. The electron oscillations are not transversal. For the small perturbations from (45) - (55) the following simple equality results:

$$\frac{\tilde{J}_{\parallel}}{\tilde{J}_{\parallel}} = \frac{3}{2} \frac{\tilde{T}_{e}}{T} = \frac{3}{8} \frac{\tilde{T}_{i}}{T} = -\frac{\tilde{v}_{eY}}{v_{d}} = +\frac{\tilde{v}_{iY}}{v_{d}} = \frac{\tilde{n}_{i}}{n} = \frac{\tilde{n}_{e}}{n} = \frac{3e\phi}{5T}$$

$$(56)$$

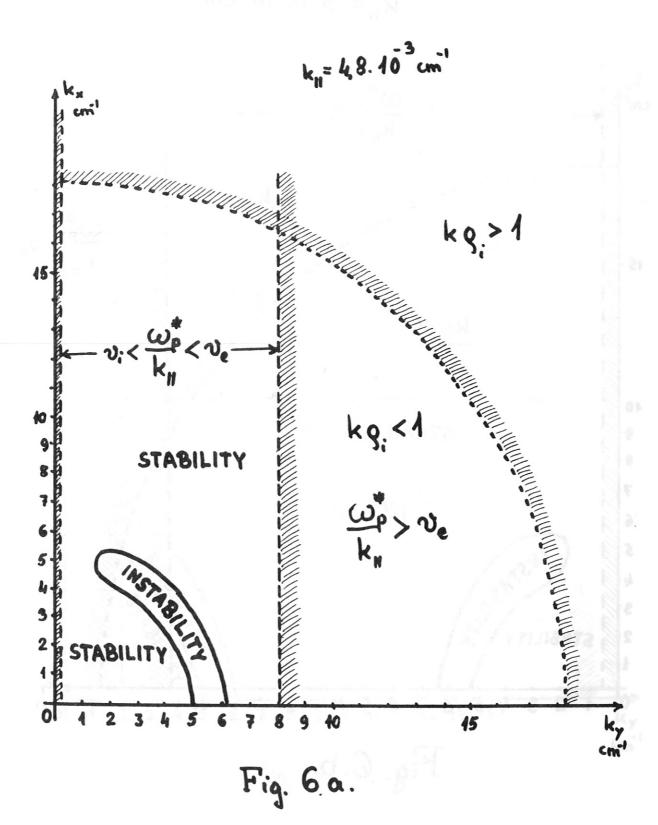
Here we must make a remark:

Of the two nonadiabatic drift waves only the electron drift waves lead to deformations of the p = const surfaces. Therefore it has to be expected that these waves are responsible for high anomalous turbulent thermal conductivity. The existence of these waves was confirmed experimentally [5]. The dominant effect is to be expected in the region of the resonance since the amplification rate is maximum. But in this case, the term 30 v_i^2 k_{\parallel}^2 of the dispersion relation (19) introduces some ionacoustic effects in the propagation of the drift waves. Therefore, it has to be expected that the ionacoustic perturbations will influence the turbulent thermal conductivity. This phenomenon was indeed observed experimentally [6].

6) Some Estimates Made from the Approximative Solution of (25)

In order to obtain an approximation for ${\rm Im}\,\omega$, we make use of ${\rm Re}\,\omega$ as determined from (20) and put this into eq. (25). We applied these approximative values to construct the stability and instability regions of the mentioned waves in the k_x - k_y plane for various k_w in the presence of both dissipative effects: the longitudinal electric resistivity and the longitudinal thermal conductivity of the electrons. We used the plasma parameters (26).

In Fig. 6a we plotted the instability regions for the case if $\mathbf{k_{N}}$ is near to the value for the first harmonics. We can see that there is a coincidence with the results plotted in Figs. 3-5. The nonadiabatic wave is unstable in the region where $\mathbf{k_{y}} < \mathbf{k_{y}}'$, and it corresponds to the stable wave if $\mathbf{k_{y}} > \mathbf{k_{y}}'$. ($\mathbf{k_{y}}'$ is the wave vector corresponding to the intersections of the dissipationfree dispersion curves). The longitudinal thermal conductivity influences the stability only in the region of small $\mathbf{k_{y}}$ and leads here to a stabilization. But the essential effect is the pronounced instability in the vicinity of the resonance, and this is determined by the electric conductivity. In Fig. 6b we represented the stability region for the second harmonics, and in Fig. 6c for the 8th harmonics. As can be seen, the shape, the location, and the dimensions of the instability



$$k_{11} = 9, 6.10^{-3} \text{ cm}^{-1}$$

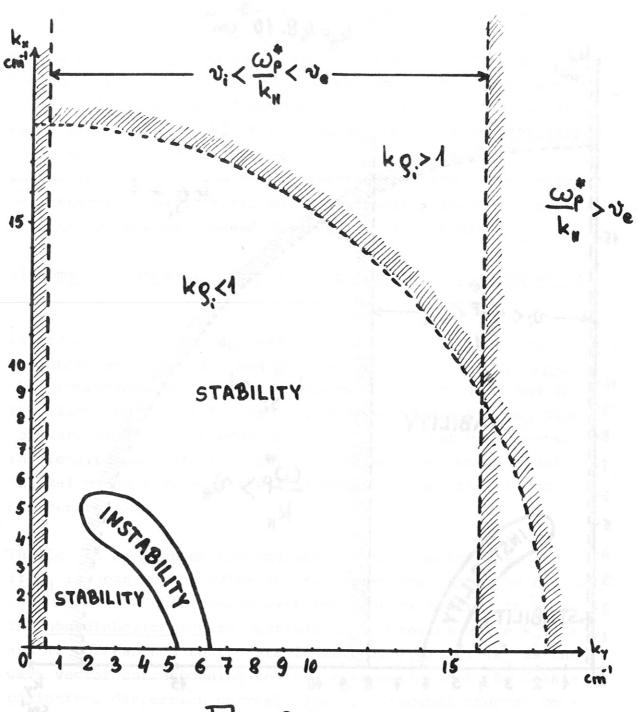


Fig. 6.b.

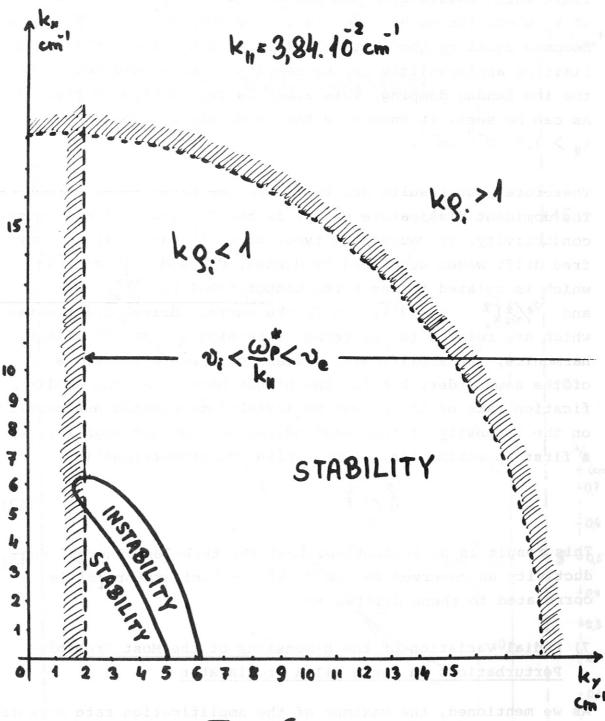


Fig. 6. c.

region are practically the same for all harmonics. The maximum of the amplification rate appears always for $\rho_i k \approx \frac{1}{3}$. The only limit which shifts from one harmonic to the other is the value of k_y where the phase velocity of the nonadiabatic drift wave becomes equal to the thermal velocity of the ions. This is a limit of applicability of our results since we did not study the ion Landau damping. This limit is represented in Figs. 6. As can be seen, it enters in the stability region if $k_y > 3.6 \cdot 10^{-2} \text{ cm}^{-1}$.

Therefore, our results are valid for the first seven harmonics. The dominant dissipative effect is the longitudinal electric conductivity. It causes two types of drift waves: 1) the current free drift waves described by Hendel, Chu and Politzer [7] which is related to the terms proportional to $\frac{\omega_p^* v_e}{v_e^* k_u^*}$ in (25), and 2) the current driven drift waves which are related to the terms containing §. For the first harmonics, the amplification rates of these two waves are of the same order, but for the higher harmonics the amplification rate of the resulting instability depends essentially on the intensity of the longitudinal current (through §). As a first approximation, we can write the proportionality

This result is an indication, that the turbulent thermal conductivity as observed in the W VII A - Stellarator [8] is correlated to these driftwaves.

7) Radial Variation of the Dimensions of the Most Probable Perturbations in the W VII-A Stellarator

As we mentioned, the maximum of the amplification rate appears for all toroidal harmonics which can be studied with our method, for

$$k_{\gamma} \approx \frac{4}{3} g_{c} \tag{58}$$

The dependence of the parameters of the most unstable current driven

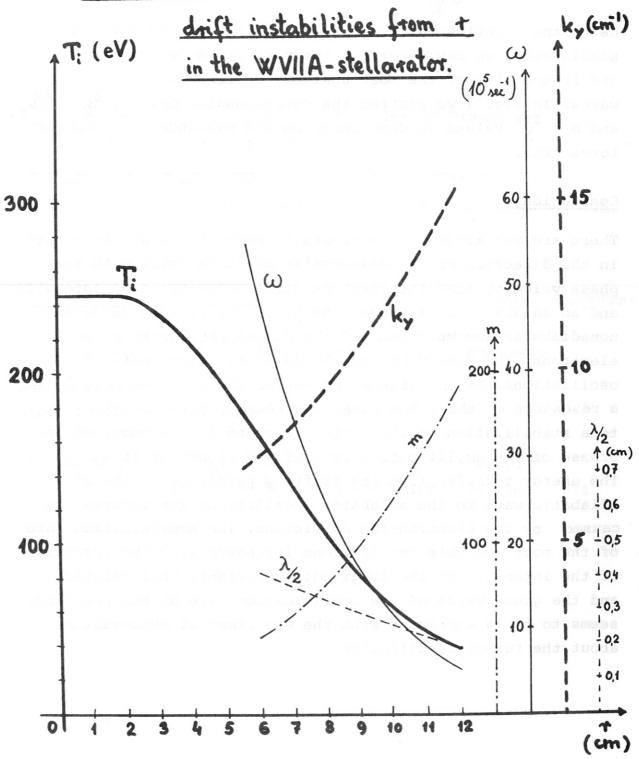


Fig. 7

The corresponding frequency is

$$\omega \approx \frac{\aleph_d}{3\varsigma_i} \tag{59}$$

Using the experimental temperature profile of the W VII-A stellarator, we can determine the most probable wavelength and frequencies of the most unstable current driven drift waves. In Fig. 7 we plotted the corresponding ω , k_y , $\frac{1}{\sqrt{2}} = \frac{\pi}{\sqrt{k_y}}$ and $m = \frac{2\pi r}{\lambda}$ values in dependence on the distance (r) from the torus axis.

Conclusions

There are two kinds of electrostatic waves which are propagating in the direction of the diamagnetic electron drift with the phase velocity near the electron drift velocity: a nonadiabatic and an adiabatic drift wave. The first one is related to the nonadiabatic accumulation and the diamagnetic drift of the electrons, the second one to the adiabatic electrostatic oscillations. As an internal effect of the inhomogeneous plasma a resonance of these two waves can appear. This resonance leads to a stabilization of the faster wave and to a pronounced increase of the amplification rate of the slower drift wave. The energy transfer from the drifting particles of the nonadiabatic wave to the adiabatic oscillations (or inverse) is caused by the electron-ion collisions. The amplification rate of the most unstable oscillations increases with the increase of the intensity of the longitudinal current. This relation and the great value of the amplification rate at the resonance seems to be in agreement with the experimental observations about the turbulent diffusion.

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